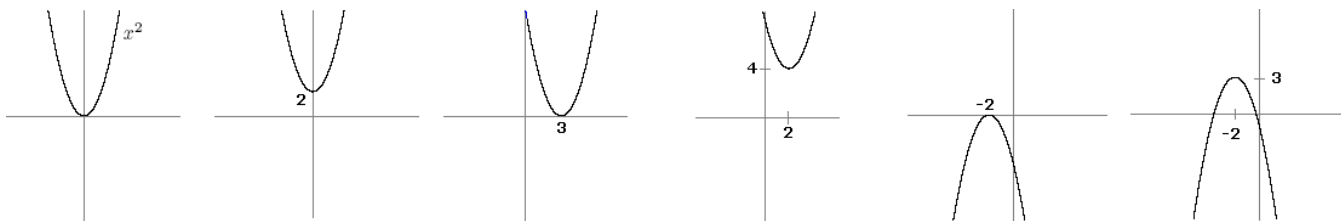


Mathematics 114 Spring 2007 – Review for Final Exam

1. Find the equation of the line passing through (1, 4) and (2, 6).
2. Find the equation of the line that is perpendicular to $y = 2x + 3$ and passes through the point (1, -4).
3. Find the domain of $f(x) = \frac{1}{x-3} + \sqrt{x-2}$.
4. Let $f(x) = x^2 + 2x + 1$ and $g(x) = e^x$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.
5. Sketch the graphs of $|x|$, $3 + |x|$, $|x - 4|$, and $3 - |x - 4|$.
6. Find the inverse of $f(x) = \frac{x^3 + 2}{3}$.
7. Sketch the graph of a function that doesn't have an inverse, and sketch the graph of a function that does have an inverse. Explain why your choices work.
8. Divide $3x^3 - 8x^2 + 6x + 9$ by $x - 2$.

9. Find the equations of the parabolas graphed below.



10. Find the zeros of the following.

(a) $f(x) = 2x^3 - 6x$ (b) $f(x) = x^2 - 5x + 6$ (c) $f(x) = 2x^2 + 5x + 7$ (d) $f(x) = \frac{x^2}{e^{x^2}}$
 (e) $f(x) = x^2 + 1$ (f) $f(x) = \sin x$ (g) $f(x) = \tan x$ (h) $f(x) = (x - 1)x^2 + 2x(x - 1)^2$

11. At what points does the graph of $f(x) = 9 - x^2$ cross the x -axis?

12. Find the horizontal and vertical asymptotes.

(a) $f(x) = \frac{2x^2 - 1}{x^2 - 9}$ (b) $f(x) = \frac{x - 1}{x^2 + 3}$ (c) $f(x) = \frac{(x - 1)(x^2 + 1)}{x - 2}$

13. Sketch the graph of a function $f(x)$ that has a horizontal asymptote at $y = 1$, vertical asymptotes at $x = -2$ and $x = 3$, and has $f(4) = 0$.

14. Simplify. (The answer to part (d) is not 7.)

(a) $\frac{e^{3x}}{e^{2x}} + (e^{x/2})^2$ (b) $e^{2 \ln x}$ (c) $e^x e^{-x} + \log_5 25$ (d) $e^{\ln 4 + \ln 3}$

15. Solve the following equations.

(a) $e^x = 8$ (b) $\ln x = 8$ (c) $2^x = 11$ (d) $e^{2x+1} - 2 = 8$ (e) $\frac{1}{1 + \ln 2x} = 3$

16. Find the inverse of $f(x) = 1 + \ln x$.

17. Write $\ln\left(\frac{x^2y^3}{z^4}\right)$ as three logarithms, and then simplify as much as possible.

18. Show that $\frac{1}{2} \ln(1-x) - \frac{1}{2} \ln(1+x) = \ln \sqrt{\frac{1-x}{1+x}}$.

19. Convert $\pi/18$ radians to degrees.

20. Find all solutions in $[0, 2\pi)$ of the following.

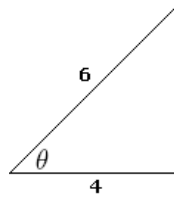
(a) $\sin x = \frac{1}{2}$ (b) $\tan 2x = 1$ (c) $\sin x + \sin x \cos x = 0$

21. Find the following. A calculator answer is not sufficient. You must provide an exact answer.

(a) $\sin 780^\circ$ (b) $\tan(-\pi/6)$ (c) $\csc 75^\circ$

22. Suppose $\sin \theta = 2/5$. Find $\sec \theta$ and θ . Assume $0 < \theta < 90^\circ$.

23. Find all 6 trigonometric functions for the angle θ , and then find $\tan 2\theta$.



24. Radium has a half-life of 1599 years. This means that after 1599 years, half of the original amount of radium will have decayed, and only half of the initial amount will remain. Suppose we start with 100 grams of radium. The amount that will be left after t years is given by

$$Q(t) = 100 \left(\frac{1}{2}\right)^{t/1599}$$

- (a) How much will remain after 200 years?
- (b) How long will it take until there is only 10 grams remaining?
- (c) Find a formula for how long it will take until there are only Q grams remaining.

25. A kite is in the air at a 70 degree angle with the ground, and 100 feet of string is out. How high is the kite?

26. Suppose you are standing 50 feet from a straight highway, and a car passes by. Three seconds later you estimate the angle between yourself and the car is 80° . How fast is the car going?

27. Verify the following identities.

(a) $\frac{\sin(x+y)}{\cos x \cos y} = \tan x + \tan y$

(b) $\tan(2\pi - \theta) = -\tan \theta$

(c) $\cot 2x = \frac{1}{2}(\cot x - \tan x)$

(d) $\sin^2 x(1 + \cot^2 x) = 1$

(e) $4 \sin^2 x + 2 \cos^2 x = 4 - 2 \cos^2 x$

(f) $\csc x(\csc x - \sin x) + \frac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$