Algebra and Trig Review (Version 2/9/07 - Brian Heinold)

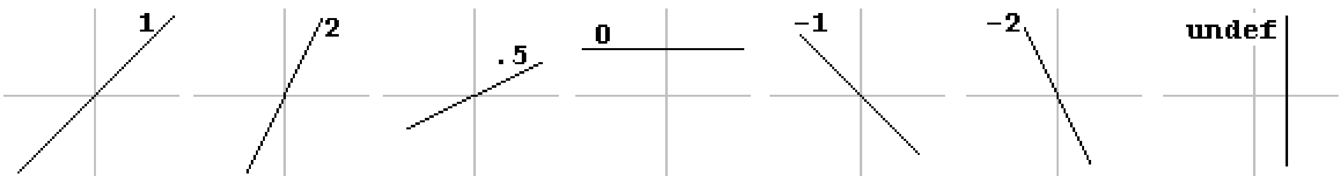
I threw this together really quickly. It certainly is not a complete list of everything that you should know, but it does cover a lot of important topics. I've mostly used examples rather than a rigorous presentation. In fact, I've tried to use plain English where possible, so please don't complain to me if I've said anything that's mathematically imprecise. This is meant as a refresher on algebra and trig, and to keep it reasonably short, I've had to skip a lot of basic topics, and had to leave out a lot of things that are useful to know. Feedback is greatly appreciated. If you see a mistake or a topic that should be covered, please do let me know.

Lines

The slope of a line is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}.$$

Below are the slopes of some lines.



There are two forms of the equation of a line that are often used.

$$y = mx + b \qquad \textit{Slope-intercept form}$$
$$y - y_1 = m(x - x_1) \qquad \textit{Point-slope form}$$

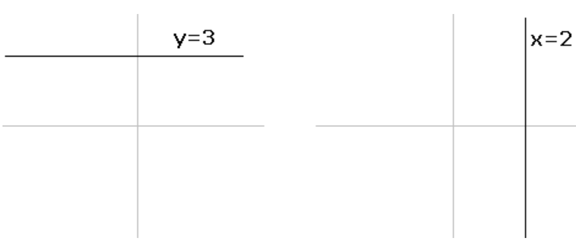
m is the slope, b is the y-intercept, which is where the line meets the y-axis, and (x_1, y_1) can be any point on the graph.

Problem: Find the equation of the line that passes through the point $(3, 4)$ and has a slope of 2.

Solution: We know $m = 2$. Using the slope-intercept form, plug in $m = 2$, $x = 3$, and $y = 4$ to solve for b. We get $4 = 2 \cdot 3 + b$, which means $b = -2$. Therefore, the equation of the line is $y = 2x - 2$.

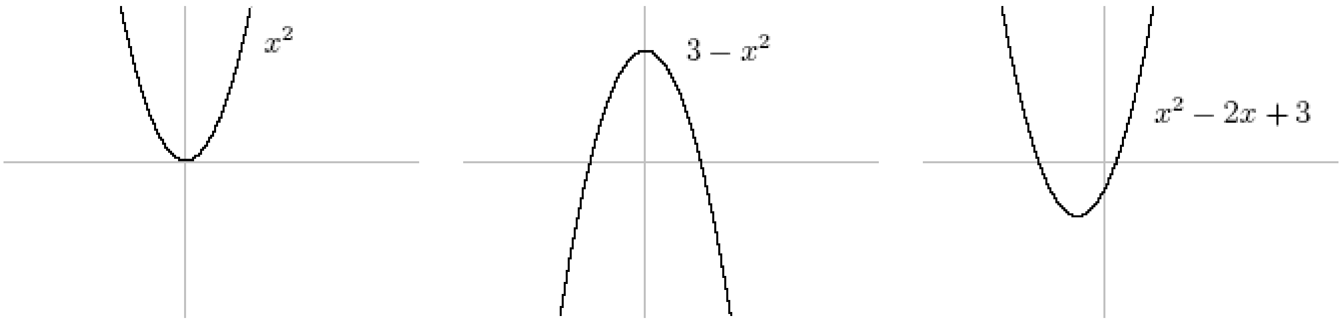
On the other hand, to use the point-slope form, we plug in $m = 2$, $x_1 = 3$, and $y_1 = 4$ to get $y - 4 = 2(x - 3)$. This is generally ok as an answer. If we simplify it, we get $y = 2x - 2$.

A horizontal line has equation $y =$ (a number), and a vertical line has equation $x =$ (a number).



Quadratic Equations

These are polynomials whose highest power is x^2. For example, $x^2 - 2x + 3$, or $3 - x^2$. Their graphs are *parabolas*.



We very often need to factor a quadratic. For example, $x^2 - 3x - 10$ factors into $(x-5)(x+2)$. The way to get this is that the two numbers must multiply together to give -10 (that is, $-5 \cdot 2 = -10$), and if we multiply the two inside terms and the two outside terms, and add them, we get $-5x + 2x = -3x$. (If the leading term is just x^2, then a short cut to this is simply that the two numbers must add up to -3, which they do, since $-5 + 2 = -3$).

As another example, $x^2 + 11x + 28$ factors into $(x+7)(x+4)$. Notice that $7 \cdot 4 = 28$, and $7 + 4 = 11$.

To solve $x^2 + 11x + 28 = 0$, factor it to get $(x+7)(x+4)$ and set each part equal to zero to get $x + 7 = 0$ and $x + 4 = 0$, so $x = -7$ and $x = -4$ are the solutions. On the other hand, we can't solve $x^2 + 7x + 3 = 0$ by factoring. In this case we can resort to the *quadratic formula.*

The solution to $ax^2 + bx + c$ is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *(Quadratic formula)*

So, to solve $x^2 + 7x + 3 = 0$, we have $a = 1$, $b = 7$, and $c = 3$, and the quadratic formula gives

$$x = \frac{-7 \pm \sqrt{49 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{-7 \pm \sqrt{37}}{2}$$

Powers

Here are some examples of rewriting powers.

$$\sqrt{x} = x^{1/2}$$
$$\sqrt[3]{x} = x^{1/3}$$
$$\sqrt[6]{x^5} = x^{5/6}$$
$$\frac{1}{x^2} = x^{-2}$$
$$\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$$
$$\frac{2}{x^4} = 2x^{-4}$$
$$\frac{3}{2x} = \frac{3}{2}x^{-1}$$
$$\frac{1}{x^{-4}} = x^4$$

In general, $\sqrt[n]{x} = x^{1/n}$, $\sqrt[n]{x^k} = x^{k/n}$, and $\dfrac{1}{x^n} = x^{-n}$. Also, $x^0 = 1$.

Also, look at the following examples.

$$\begin{aligned}
x^3x^8 &= x^{11} \\
\frac{x^9}{x^4} &= x^5 \\
\frac{x^3}{x^{10}} &= \frac{1}{x^7} \text{ or } x^{-7} \\
(x^2)^7 &= x^{14}
\end{aligned}$$

In general, when multiplying, we add exponents, when dividing, we subtract them, and if we're raising a power to a power, then we multiply the exponents.

Solving Equations

- For a simple equation like $1 - 2x = 7$, do the addition or subtraction first, and then do any multiplying or dividing.

$$\begin{aligned}
1 - 2x &= 7 \\
-2x &= 6 \\
x &= -3
\end{aligned}$$

- For the equation the equation $1 + \frac{x^2}{3} = 8$. First do the subtraction, then the multiplication, and take the square root last of all. Doing things in this order is often helpful.

$$\begin{aligned}
1 + \frac{x^2}{3} &= 8 \\
\frac{x^2}{3} &= 7 \\
x^2 &= 21 \\
x &= \pm\sqrt{21}
\end{aligned}$$

  Note that when taking the square root, a $\pm$ is needed. It is also needed when taking a fourth root, sixth root, or any even root. It is not needed for odd roots.

- We won't be using any imaginary numbers in this class, so that an equation like $x^2 + 9 = 0$ will have *no solution.*

- Try to factor equations as much as possible. For example, consider $2x^3 - 4x = 0$.

$$\begin{aligned}
2x^3 + 4x &= 0 \\
2x(x^2 - 4) &= 0 \\
2x(x-2)(x+2) &= 0
\end{aligned}$$

  At this point, set each of the factors equal to 0 separately. Setting the first equal to 0, we get $2x = 0$, so $x = 0$ is one solution. The second gives $x = 2$, and the third gives $x = -2$. (Note that we could have stopped after the second step and solved $x^2 - 4 = 0$, instead.)

Warning! Setting each term equal to 0 separately only works if the terms are all *multiplied* together. If they are added together, then it won't work.

Warning! This technique only works if right side is 0. If it is anything else, it won't work! The reason this method works at all is because if $ab = 0$, then either $a = 0$ or $b = 0$. However if $ab = 4$, for example, it is no longer necessarily true that either $a = 4$ or $b = 4$.

- Here's another example.

$$\begin{aligned} 15x^2(x-1)^2 + 10x^3(x-1) &= 0 \\ 5x^2(x-1)\big(3(x-1)+2x\big) &= 0 \\ 5x^2(x-1)(3x-3+2x) &= 0 \\ 5x^2(x-1)(5x-3) &= 0 \end{aligned}$$

In the second step we factored out a $5x^2(x-1)$. Why? Well, there's two terms added together, and we examine them to see what each one has in common – each is a multiple of 5, the first has an x^2 and the second an x^3, so they have an x^2 in common, and they also have an $(x-1)$ in common. The right-hand parentheses contains whatever is left over after factoring. Once we've finished factoring, we simplify things and finally, to finish the problem, set $5x^2 = 0$ to get $x = 0$, set $x - 1 = 0$ to get $x = 1$, and set $5x - 3 = 0$ to get $x = 3/5$.

- An example with a fraction: Multiply both sides by the denominator to clear the fractions.

$$\begin{aligned} \frac{1}{2x} &= 4 \\ 1 &= 8x \\ \frac{1}{8} &= x \end{aligned}$$

- If we set a fraction equal to 0, we still multiply both sides by the denominator to clear the fractions. Notice what happens here, the denominator disappears from the problem, since it gets multiplied by 0. To summarize: *If we're setting a fraction equal to 0, we just have to set the numerator equal to 0.*

$$\begin{aligned} \frac{x+4}{x^3+3} &= 0 \\ x+4 &= 0 \\ x &= -4 \end{aligned}$$

- A harder example: Solve $(x+4)^3(3+y)+1 = 2x$ for y. We want to get y by itself on the left side. Here's where dividing by $(x+4)^3$ too soon could cause a problem if we're not careful. The safest bet is first to subtract 1 from each side.

$$\begin{aligned} (x+4)^3(3+y)+1 &= 2x \\ (x+4)^3(3+y) &= 2x-1 \\ 3+y &= \frac{2x-1}{(x+4)^3} \\ y &= \frac{2x-1}{(x+4)^3} - 3 \end{aligned}$$

- Solve $(x^2-9)^2=0$. Don't expand the left side, as it will give $x^4-18x^2+81=0$, and in that form, it's not clear at all what to do. Rather, take the square root of both sides.

$$\begin{aligned}(x^2-9)^2&=0\\ x^2-9&=0\\ x^2&=9\\ x&=\pm 3\end{aligned}$$

- Solve $2x-\dfrac{200}{x}=0$. It's probably best to move the fraction to the other side, and then clear the fractions.

$$\begin{aligned}2x-\frac{200}{x}&=0\\ 2x&=\frac{200}{x}\\ 2x^2&=200\\ x^2&=100\\ x&=\pm 10\end{aligned}$$

- Solve $\dfrac{1}{\sqrt[3]{x-1}}+2=y$ for x.

$$\begin{aligned}\frac{1}{\sqrt[3]{x-1}}+2&=y\\ \frac{1}{\sqrt[3]{x-1}}&=y-2\\ 1&=(y-2)\sqrt[3]{x-1}\\ \frac{1}{y-2}&=\sqrt[3]{x-1}\\ \left(\frac{1}{y-2}\right)^3&=x-1\\ \left(\frac{1}{y-2}\right)^3+1&=x\end{aligned}$$

- Solve $2x=x^2$ for x.

$$\begin{aligned}2x&=x^2\\ 2x-x^2&=0\\ x(2-x)&=0\end{aligned}$$

Therefore, the solutions are $x=0$ and $x=2$. I put this here because a common mistake would be to right away divide by x. The problem with that is then we'd miss the solution $x=0$. So the safer thing to do is move everything to one side and then factor.

Fractions

First, here's three examples of how to simplify a complex fraction. Just multiply top and bottom by the reciprocal of the bottom.

$$\frac{\frac{x^2}{4}}{\frac{\cancel{y}}{\cancel{3}}} \cdot \frac{\frac{3}{y}}{\frac{\cancel{3}}{\cancel{y}}} = \frac{3x^2}{4y}$$

$$\frac{\frac{x^2}{4}}{\cancel{7}} \cdot \frac{\frac{1}{7}}{\frac{1}{\cancel{7}}} = \frac{x^2}{28}$$

$$\frac{x^2}{\frac{\cancel{y}}{\cancel{3}}} \cdot \frac{\frac{3}{y}}{\frac{\cancel{3}}{\cancel{y}}} = \frac{3x^2}{y}$$

It is sometimes convenient to break up a fraction into two fractions. For example,

$$\frac{x^2+4}{y} = \frac{x^2}{y} + \frac{4}{y}$$

We will sometimes need to add or subtract fractions. To get a common denominator, multiply each fraction by the other's denominator. For example, suppose we want to add $\frac{2}{x} + \frac{x^2}{y}$.

$$\frac{2}{x} \cdot \frac{y}{y} + \frac{x^2}{y} \cdot \frac{x}{x} = \frac{2y}{xy} + \frac{x^3}{xy} = \frac{2y+x^3}{xy}$$

As another example, suppose we have to solve $\frac{1}{x} + \frac{x}{2} = 3$. Start by finding a common denominator on the left side.

$$\begin{aligned}
\frac{1}{x} \cdot \frac{2}{2} + \frac{x}{2} \cdot \frac{x}{x} &= 3 \\
\frac{2}{2x} + \frac{x^2}{2x} &= 3 \\
\frac{2+x^2}{2x} &= 3 \\
2 + x^2 &= 6x \\
x^2 - 6x + 2 &= 0 \\
x &= \frac{6 \pm \sqrt{28}}{2} \qquad \text{(quadratic formula)}
\end{aligned}$$

Canceling

We have to be quite careful about canceling things out. Essentially, when you cancel out a term, you are erasing it from existence – not a thing to be taken lightly!

In the example below, we can cancel out the x^3's.

$$\cancel{x}^3 + x^2 + 7 - \cancel{x}^3$$

However, in the this next example, they can't be canceled out, since the 3 gets in the way.

$$x^3 + 3(x - x^3) \quad \text{(no cancelation possible)}$$

In the example below, we can cancel out the x^3's.

$$\frac{\cancel{x}^3(x-2)}{\cancel{x}^3(2y+1)}$$

Notice that the terms are *multiplied together*. However, if the terms were added or subtracted, then we couldn't cancel.

$$\frac{x^3 + (x-2)}{x^3 + (2y+1)} \quad \text{(no cancelation possible)}$$

In the example below, we can cancel out the x^3's because they're both in the denominator of their respective fractions.

$$\frac{\frac{3}{\cancel{x}^3}}{\frac{y}{\cancel{x}^3}} = \frac{3}{y}$$

It works if they're both in the numerator, too.

$$\frac{\frac{\cancel{x}^3}{3}}{\frac{\cancel{x}^3}{y}} = \frac{\frac{1}{3}}{\frac{1}{y}} = \frac{y}{3}$$

However, if one is in the numerator, and the other in the denominator, then things won't work out.

$$\frac{\frac{x^3}{3}}{\frac{y}{x^3}} \quad \text{(no cancelation possible)}$$

A safe way to cancel terms is to factor first. Consider the example below.

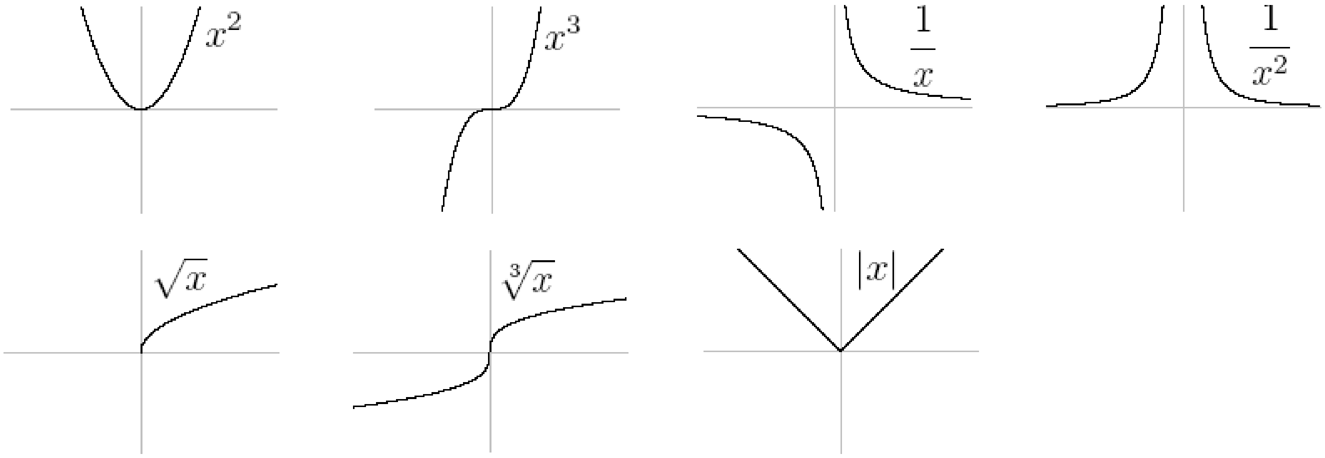
$$\frac{3x^2 + 6x^4}{9x + 3x^3} = \frac{3x^2(1+2x^2)}{3x(3+x^2)} = \frac{x(1+2x^2)}{3+x^2}$$

The top and bottom have a $3x$ in common, and canceling this out leaves an extra x in the numerator.

Finally, in the case of $\frac{x+1}{x}$, no cancelation is possible, but the expression can be rewritten as $\frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x}$, which may be useful for certain problems.

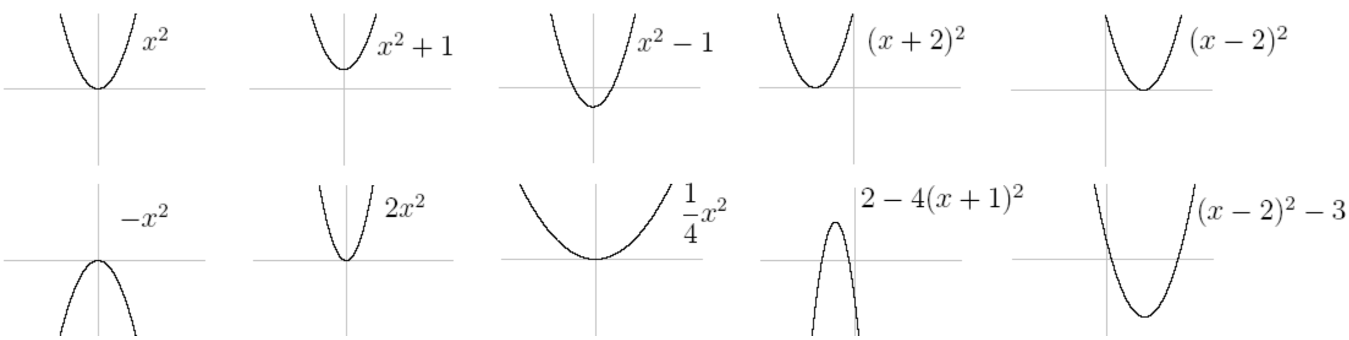
Some Common Graphs

Below are some graphs which are really useful to know. If you know these, you automatically know a bunch more. For instance x^4, x^6, x^8, etc. look a lot like x^2, and the odd powers look a lot like x^3. Moreover, even roots $\sqrt[4]{x}$, $\sqrt[6]{x}$, $\sqrt[8]{x}$, etc. look a lot like $\sqrt{x}$, and odd roots look a lot like $\sqrt[3]{x}$. Finally, $1/x^4$, $1/x^6$, etc. look a lot like $1/x^2$, and the odd powers look a lot like $1/x$.



Graph Transformations

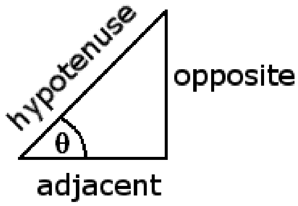
By example: adding 1 to the function on the outside, as in $f(x)+1$, shifts the graph up by 1, whereas subtracting 1 shifts it down by 1. Adding 2 to the inside of the function, as in $f(x+2)$, shifts the graph *left* by 2, whereas subtracting 2 shifts it right by 2. A minus on the outside, $-f(x)$, flips the graph across the x-axis. A minus on the inside, $f(-x)$, reflects it across the y-axis. Multiplying the function by 2 often makes it more narrow, and multiplying by 1/4 makes it wider. We can combine several of these at once.

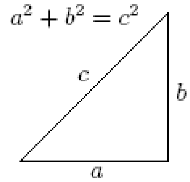


To determine what the graph of $(x-2)^2+3$ looks like, we recognize that it comes from x^2, with the $+3$ on the outside shifting it up by 3, and the -2 on the inside shifting it right by 3. Similarly, $2-4(x+1)^2$ is gotten from x^2 by shifting up 2 because of the 2 on the outside, shifted left by 1 because of the $+1$ on the inside, flipped because of the minus sign, and narrowed because of the 4.

Trigonometry

The three sides of a right triangle are often called the opposite (opposite the angle), the adjacent (next to the angle), and the hypotenuse.



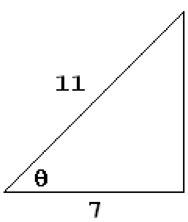


If we abbreviate these by o, a, and h, we can write the trig functions $\sin\theta$, $\cos\theta$ and $\tan\theta$ as

$$\sin\theta = \frac{o}{h} \quad \cos\theta = \frac{a}{h} \quad \tan\theta = \frac{o}{a}$$

A useful way to remember this is "sohcahtoa" (pronounce it as sō-ca-tō-a). Important also is the Pythagorean Theorem, $a^2 + b^2 = c^2$. There are three other trig functions that are often used, which are the reciprocals of the main functions: $\csc\theta$ is the reciprocal of $\sin\theta$, $\sec\theta$ is the reciprocal of $\cos\theta$, and $\cot\theta$ is the reciprocal of $\tan\theta$.

Problem: For the triangle below, find all six trig functions.



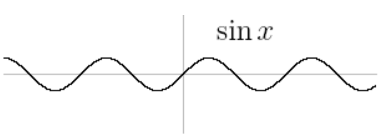
Solution: First, use the Pythagorean Theorem to find the missing side — solve $7^2 + b^2 = 11^2$ for b to get $b = \sqrt{62}$. Then we get

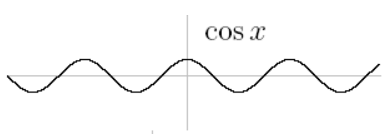
$$\sin\theta = \frac{\sqrt{62}}{11} \qquad \csc\theta = \frac{11}{\sqrt{62}}$$

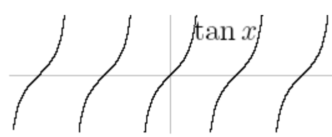
$$\sec\theta = \frac{7}{11} \qquad \sec\theta = \frac{11}{7}$$

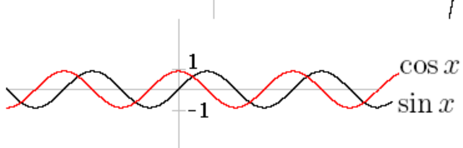
$$\tan\theta = \frac{\sqrt{62}}{7} \qquad \cot\theta = \frac{7}{\sqrt{62}}$$

Below are the graphs of the three main trig functions. Each is *periodic*, meaning they keep repeating the same pattern forever. The peaks of $\sin x$ and $\cos x$ are at 1, and the valleys are at -1. The shapes of $\sin x$ and $\cos x$ are identical, they're just shifted versions of each other. Use the bottom graph to see the difference between $\sin x$ and $\cos x$ ($\sin x$ is the one that passes through the origin $(0,0)$).









Trig Identities

What follows below are what I think are the most useful trig identities. These are extremely useful. There are tons of other identities, if you're interested, and the inside cover of your book is a good place to find more of them.

Converting to sin and cos

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

Reciprocal

$$\frac{1}{\tan x} = \cot x \qquad \frac{1}{\sec x} = \cos x$$

$$\frac{1}{\cot x} = \tan x \qquad \frac{1}{\csc x} = \sin x$$

Pythagorean

$$\sin^2 x + \cos^2 x = 1 \qquad 1 - \cos^2 x = \sin^2 x$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 - \sin^2 x = \cos^2 x$$

Radians and Degrees

It's more comfortable to measure angles in degrees, but mathematically it turns out to be much more convenient to use *radians*. You can use the following to convert between the two.

Degrees to radians — multiply by $\frac{\pi}{180}$ Radians to degrees — multiply by $\frac{180}{\pi}$

Some common angles that are good to know at the drop of a hat are

$0°$ —— 0 rad

$30°$ —— $\frac{\pi}{6}$ rad

$45°$ —— $\frac{\pi}{4}$ rad

$60°$ —— $\frac{\pi}{3}$ rad

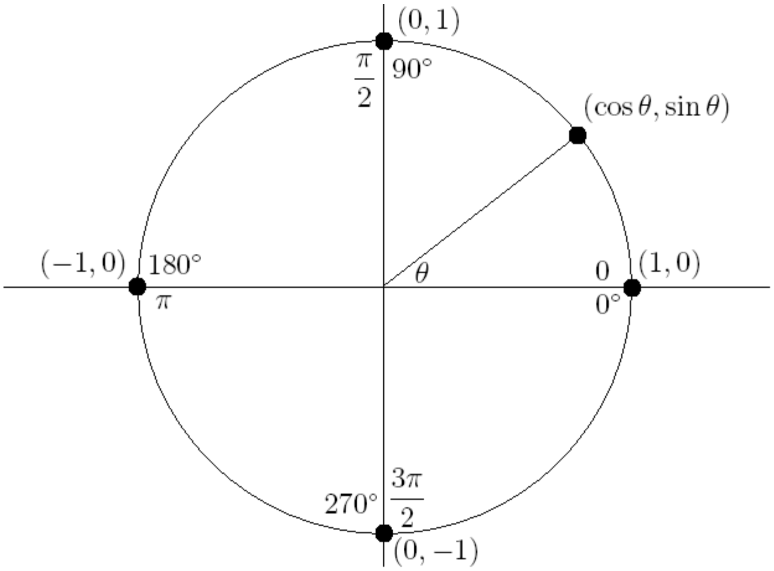
$90°$ —— $\frac{\pi}{2}$ rad

$180°$ —— π rad

$270°$ —— $\frac{3\pi}{2}$ rad

$360°$ —— 2π rad

The Unit Circle



One trip all the way around is 360°, or 2π radians. Two trips all the way around are 720°, or 4π radians. Often angles are measured backwards, so that, for example, 270° is also called −90°. The trig functions $\cos\theta$ and $\sin\theta$ give the coordinates of the point on the unit circle at the angle θ. This gives a nice way to remember the values of sin and cos at angles which are multiples of π or $\pi/2$, which are the most useful ones to know. Remember that

$\cos\theta$ gives the x-coordinate
$\sin\theta$ gives the y-coordinate

So, for example, to find $\cos\pi$ and $\sin\pi$, find π on the unit circle. Since π radians is 180°, it is at the point $(-1, 0)$. Therefore, $\cos\pi = -1$, and $\sin\pi = 0$. As another example, $\cos 8\pi = 1$ and $\sin 8\pi = 0$, since 8π radians is in the same place as 0 radians, namely at the point $(1, 0)$.

Miscellaneous

- *Warning!* You will never see sin by itself without an argument. It always has to have an argument, like $\sin x$, or $\sin\pi$. Moreover, it can never be separated from it argument. Despite the fact that $\sin(x^2+1)$ looks like multiplication, it really isn't. Attempting to separate the sin from its argument will cause you to incur the wrath of your professor, something you must certainly avoid. Here's some examples:

  In the example below on the left, the $\sin(x^2+1)$ term can be canceled, since the arguments of sin are both the same. However, in the example on the right, the arguments are different, and so no cancelation can be done.

  $$\frac{7x\sin(x^2+1)}{2y\sin(x^2+1)} = \frac{7x}{2y} \qquad\qquad \frac{7x\sin(x^2+1)}{\sin(2x^2+2)} \quad \text{(no cancelation possible)}$$

  Also, though it might make life easier, there is no simple way to rewrite $\sin(x+y)$. The simplest way is that it can be rewritten using the trig identity

  $$\sin(x+y) = \sin x\cos y + \cos x\sin y.$$

- A very common mistake is to mess up the distributive rule. For example, be extra careful when simplifying the expression $4(x^2 + 2x + 8)$. Be sure to make sure you have multiplied the 4 by every term. You should get $4x^2 + 8x + 32$.

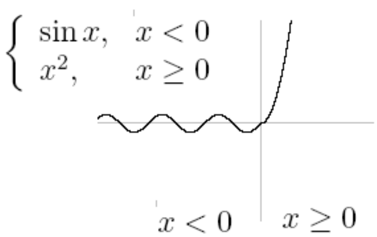
  As an other example, an extremely easy mistake is to forget to apply the minus sign to every term. For example $x^2 - (3x + 4y - 12)$ becomes $x^2 - 3x - 4y + 12$. Always be extra careful when simplifying things like this.

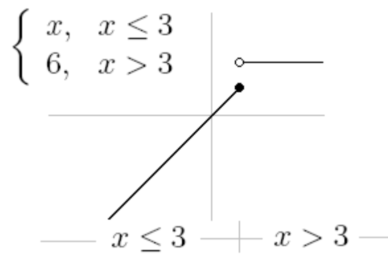
  Also, the distributive rule works if the thing being multiplied is on the right side. For example, $(x^2 - 2x + 4) \cdot 5 = 5x^2 - 10x + 20$.

- How could we solve the equation $\sin x = 3$? Answer – we can't. To see why, look at the graph of $\sin x$. It never gets above 1, so $\sin x = 3$ has no solution.

- **Piecewise functions**

  The graphs shown below are called *piecewise functions*.





  Let's use second function as an example; call it $f(x)$. The function does $f(x) = x$ up until $x = 3$ and then switches to $f(x) = 6$. At $x = 3$, there's a closed circle and an open circle. We use a closed circle on the $f(x) = x$ part of the graph because that equation if valid for $x \leq 3$ (less than *or equal to* 3), and we use an open circle on the $f(x) = 6$ part of the graph because that equation is only valid for values of x *larger than* 3.

- Expand $(x + y)^2$. We do this by FOIL: $(x + y)(x + y) = x^2 + 2xy + y^2$. A common mistake is to forget the middle term.

- Factor $x^2 - y^2$. This factors into $(x - y)(x + y)$. This shows up all over the place. Here are some other examples:

$$\begin{aligned} x^2 - 16 &= (x - 4)(x + 4) \\ x^4 - 81 &= (x^2 - 9)(x^2 + 9) \\ 4y^2 - 25x^2 &= (2y - 5x)(2y + 5x) \end{aligned}$$

- Notice how $7(x - 3) + 14(x - 3)^2$ factors into $7(x - 3)\big(1 + 2(x - 3)\big)$. The entire first term was factored out, but it doesn't totally disappear; we still need a 1 there.

- Simplify $(x^2 - 9)(2x) - (x^2 + 1)(2x)$. This is a typical sort of thing. One way is to distribute and then combine like terms. Be careful with the signs.

$$\begin{aligned} &(x^2 - 9)(2x) - (x^2 + 1)(2x) \\ &= 2x^3 - 18x - (2x^3 + 2x) \\ &= 2x^3 - 18x - 2x^3 - 2x \\ &= -20x \end{aligned}$$

- Here's a typical example of simplifying a complicated expression.

$$
\begin{aligned}
&\frac{(x^2-9)^2\cdot 20-(-20x)\cdot 2(x^2-9)(2x)}{(x^2-9)^4}\\
&=\frac{20(x^2-9)^2-80x^2(x^2-9)}{(x^2-9)^4}\\
&=\frac{20(x^2-9)\left[(x^2-9)-4x^2\right]}{(x^2-9)^4}\\
&=\frac{20(-3x^2-9)}{(x^2-9)^3}\\
&=\frac{-60(x^2-3)}{(x^2-9)^3}
\end{aligned}
$$

Take some time to work through this. The first simplification is to rewrite the terms in a little more friendly form. After that we look at each of the two terms in the numerator, and see what they have in common. They have a 20 and an (x^2-9) in common, so that is what we factor out. The terms in the square bracket are what is left from factoring. The (x^2-9) is what is left over from the $20(x^2-9)^2$ term, and the $-4x^2$ is what is left over from the $80x^2(x^2-9)$ term. After factoring, we see that the (x^2-9) in the numerator can cancel with one of the (x^2-9)'s in the denominator. Other than that there's some combining of like terms, and a little more factoring at the end to make things look nicer.

- The three terms below are all the same thing, just written differently. It is good to be comfortable moving between the different forms.

$$
2\frac{x}{3} \qquad \frac{2x}{3} \qquad \frac{2}{3}x
$$

- Here's another typical thing to simplify.

$$
\begin{aligned}
&x^2\left(\frac{1200-2x}{4x}\right)\\
&x\left(\frac{1200-2x}{4}\right)\\
&x\left(\frac{1200}{4}-\frac{2x}{4}\right)\\
&x(300-\frac{1}{2}x)\\
&300x-\frac{1}{2}x^2
\end{aligned}
$$

It is quite useful to be able to rewrite things like this in calculus. It makes taking derivatives much easier. To summarize the steps: We first can cancel out an x. After that, breaking up the fraction turns out to be a good idea. From there the simplifications are fairly straightforward.

- A square root (or any power) can be broken up.

$$\sqrt{xy} = \sqrt{x}\sqrt{y}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

$$(xy)^{-5} = x^{-5}y^{-5}$$

$$\left(\frac{x}{y}\right)^7 = \frac{x^7}{y^7}$$

  However, addition and subtraction can't be broken up. For instance, there's no easy way to break up $\sqrt{x+y}$.