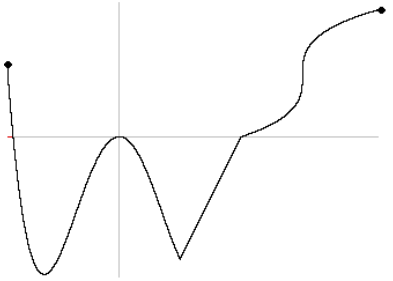
Mathematics 247 Spring 2007 – Review for Test 2

1. On the graph given below, indicate the absolute extrema, relative extrema, and critical points.



2. Find the critical numbers of the following functions.

   (a) $f(x) = x^3 - 6x^2 + 15$ (b) $f(x) = \dfrac{x+3}{x^2}$ (d) $f(x) = (x-1)^{2/3} - \dfrac{2}{3}x$

3. Determine the following limits.

   (a) $\lim\limits_{x\to\infty} x^2 + 4$ (b) $\lim\limits_{x\to-\infty} 3 - \dfrac{1}{x} - \dfrac{3}{x^2}$ (c) $\lim\limits_{x\to\infty} 2 + \sqrt{4 - \dfrac{1}{x^3}}$

   (d) $\lim\limits_{x\to-\infty} \dfrac{2x - 3x^4 + 5}{4x^4 + 8x^3 + 9x + 2}$ (e) $\lim\limits_{x\to-\infty} \dfrac{2x^5 + 150}{x^8 + 4x^2 + 2}$ (f) $\lim\limits_{x\to\infty} \dfrac{x^2 - 3x}{4x + 5}$

   (g) $\lim\limits_{x\to\infty} \cos\left(1 - \dfrac{1}{x^2}\right)$ (h) $\lim\limits_{x\to\infty} \dfrac{\cos x}{x}$ (i) $\lim\limits_{x\to\infty} \cos x$

4. Find the absolute maximum and minimum of the following functions on the given intervals.

   (a) $f(x) = x^3 - 12x$ on [0,4] (b) $f(x) = 3 - |x - 3|$ on [-1,5]

5. For parts (a) and (b), find (i) the horizontal and vertical asymptotes (if any), (ii) the intervals of increase and decrease and relative extrema, and (iii) the intervals of concavity and inflection. Use this information to sketch the graphs.

   (a) $f(x) = 3x^4 + 4x^3$

   (b) $f(x) = \dfrac{2x}{x^2 - 1}$, given that $f'(x) = \dfrac{-2(x^2+1)}{(x^2-1)^2}$ and $f''(x) = \dfrac{4x(x^2+3)}{(x^2-1)^3}$.

6. Sketch the graphs of $f(x) = |x^2 - 4|$ and $g(x) = |\sin x|$. Find the values at which $f(x)$ and $g(x)$ are not differentiable.

7. (a) Find the equation of a function $f(x)$ with vertical asymptotes at $x = \pm 3$ and a horizontal asymptote at $y = 4$.

   (b) Find the equation of a function $f(x)$ with vertical asymptotes at $x = \pm 3$, a horizontal asymptote at $y = 4$, and $f(6) = 0$.

8. Sketch the graph of any function with horizontal asymptotes at $y = \pm 1$ and vertical asymptotes at $x = \pm 2$. (*Hint: you can cross horizontal asymptote near the middle of a graph, but you can never cross a vertical asymptote.*)

9. Sketch the graph of a single function whose limit as x approaches ∞ is $-\infty$, whose limit as x approaches $-\infty$ is undefined, and whose limit as x approaches 3 is ∞.

10. Sketch the graph of a single function which satisfies all of the following:

    - It has a horizontal asymptote at $y = 1$ and vertical asymptotes at $x = \pm 3$.
    - It has a relative maximum at the point $(0, -1)$.
    - It has a relative minimum at the point $(2, -4)$.
    - It has an inflection point at the point $(1, -2)$.
    - It is increasing on the intervals $(-\infty, -3)$, $(-3, 0)$, $(2, 3)$, and $(3, \infty)$, and is decreasing everywhere else.
    - It is concave up on the intervals $(-\infty, -3)$ and $(1, 3)$, and concave down everywhere else.

11. Use Newton's Method to approximate the real zero of the function $x^5 + x - 1$. Use the initial value $x_1 = 1$, and perform three iterations.

12. Explain why Newton's Method fails to find a zero of $x^2 - 4x - 1$ if the initial value $x_1 = 2$ is used.

13. Use differentials or a linear approximation to approximate $\sqrt[3]{8.12}$.

14. Find the linear approximation of $f(x) = \sqrt{x}$ near $x = 16$.

15. Suppose that the radius of a spherical object is measured to be 16 cm with a potential error of .5 cm. Use differentials to estimate the error in the calculated volume of the sphere. Then find the percent error. (*Hint:* $V = \frac{4}{3}\pi r^3$)

16. (a) Find the dimensions of a rectangle with area 100 ft^2 whose perimeter is as small as possible.
    (b) Find the dimensions of a rectangle with perimeter 100 ft whose area is as large as possible.

17. A box with a square base and an *open top* is to be made from 600 in^2 of material. Find the largest possible volume of such a box.

18. The electric power P in watts in a direct-current circuit with two resistors R_1 and R_2 connected in parallel is

$$P = \frac{vR_1R_2}{(R_1 + R_2)^2}$$

    where v is the voltage. If v and R_1 are held constant, what resistance R_2 produces maximum power? (*Hint: the variable is in the problem is R_2. It might help to rename it x.*)

19. Find the absolute maximum and minimum of the following functions on the given intervals, or explain why there isn't any.

    (a) $f(x) = x^2$ on [-1,2] (b) $f(x) = x^2$ on (-1,2) (c) $f(x) = x^2 - 6x + 8$ on $(-\infty, \infty)$

20. Given that the inflection points of $f(x)$ are located at $x = 0$, π, and $-\pi$, where must the inflection points of $g(x) = f(x - 3)$ and $h(x) = f(x) + 3$ be located? (*Hint: how do the graphs of $g(x)$ and $h(x)$ compare with the graph of $f(x)$?*)

21. For each of the following questions, either reason it out, or use the Mean Value Theorem.

    (a) If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?
    (b) Does there exist a continuous and differentiable function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x? Explain.