Rubik’s Cubes
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A variety of cubes

1974 Ernő Rubik

debuted in 1980
The 15-Puzzle

Why it can’t be solved

Swapping 14 and 15 is an *odd permutation*. But it takes an even number of permutations to move the blank square away from and eventually back to the bottom right corner. This can be made precise using abstract algebra.
Impossible Cube
Counting

Number of states: $8! \cdot 3^7 \cdot 12! \cdot 2^{11} \approx 4.3 \times 10^{19}$.

If you disassemble and randomly reassemble the cube, there is a 1 in 12 chance that it will be solvable.

If you peel off the stickers and randomly replace them, the probability that the new cube will be solvable is

$$\frac{8! \cdot 3^7 \cdot 12! \cdot 2^{11}}{(6! \cdot 9! \cdot 6)} \approx 3.1 \times 10^{-16}.$$
Counting

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\[
\frac{8! \cdot 3^7 \cdot 12! \cdot 2^{11}}{54!/(6! \cdot (9!)^6)} \approx 3.1 \times 10^{-16}.
\]
No matter how scrambled the cube is, you never need more than 20 moves to solve it.
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See www.cube20.org for more.
Superflip
### Records

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World record video

Slow motion
How I learned to solve the cube


1. Build a $2 \times 2 \times 2$ corner
2. Expand to $2 \times 2 \times 3$
3. Twist the edges
4. Finish 2 layers
5. Position the corners
6. Twist the corners
7. Position the edges
Corners
2 × 2 × 2 Corner
$2 \times 2 \times 3$
“Twist the edges”
“Finish 2 layers”
“Finish 2 layers”
“Finish 2 layers”
Algorithmic Interlude...
“Position the corners”
“Twist the corners”
“Position the edges”
Done!


